

Exercise 31

For the following exercises, given each set of information, find a linear equation satisfying the conditions, if possible.

$$f(-1) = 4 \text{ and } f(5) = 1$$

Solution

The general formula for the equation of a line is

$$y = mx + b.$$

The first condition says that when $x = -1$, $y = 4$.

$$4 = m(-1) + b$$

The second condition says that when $x = 5$, $y = 1$.

$$1 = m(5) + b$$

This is a system of two equations with two unknowns that can be solved.

$$\begin{cases} -m + b = 4 \\ 5m + b = 1 \end{cases}$$

Subtract the respective sides of these two equations to eliminate b .

$$-m - 5m = 4 - 1 \quad \rightarrow \quad -6m = 3 \quad \rightarrow \quad m = -\frac{1}{2}$$

Multiply both sides of the first equation by 5

$$\begin{cases} -5m + 5b = 20 \\ 5m + b = 1 \end{cases}$$

and then add the respective sides of these two equations to eliminate m .

$$5b + b = 20 + 1 \quad \rightarrow \quad 6b = 21 \quad \rightarrow \quad b = \frac{7}{2}$$

Now that m and b are solved for, the equation of the line is known.

$$y = -\frac{1}{2}x + \frac{7}{2}$$